

Floquet theorem for open systems and its applications

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For a closed system with periodic driving, Floquet theorem tells that the time evolution operator can be written as $U(t, 0) \equiv P(t)e^{\frac{i}{\hbar}H_F t}$ with $P(t+T) = P(t)$, and H_F is Hermitian and time-independent called Floquet Hamiltonian. In this work, we extend the Floquet theorem from closed systems to open systems described by a Lindblad master equation that is periodic in time. Lindbladian expansion in powers of $\frac{1}{\omega}$ is derived, where ω is the driving frequency. Two examples are presented to illustrate the theory. We find that appropriate trace preserving time-independent Lindbladian of such a periodically driven system can be constructed by the application of open system Floquet theory, and it agrees well with the exact dynamics in the high frequency limit.

I. INTRODUCTION

Periodically driven quantum dynamics has recently attracted much attention both in experiment and theory[1–13], as it possesses novel properties such as topological phases[14–16] and quantum phase transitions[17, 18] that otherwise would be impossible to achieve in the undriven case. Therefore, the application of ac fields has become a very promising tool to engineer quantum systems.

It is difficult to handle a quantum system driven periodically by a field with respect to its undriven counterpart, fortunately, Floquet theorem provides us with a method to deal with such a system. It tells that for a closed system governed by a periodic Hamiltonian, its evolution operator can be decomposed into two parts, one can be given by a time-independent effective Hamiltonian (called Floquet Hamiltonian) and another is periodic in time describing the periodic micromotion of the driven system (called micromotion operator). Hence, we can design a suitable time-periodic driving to change the properties of the Floquet Hamiltonian as well as the micromotion operator. This method has been used in many experiments with ultracold atoms in driven optical lattices, for example, the dynamic localization [19–26], the control of the bosonic superfluid-to-Mott-insulator transition [27, 28], resonant coupling of Bloch bands [29–32], the dynamic creation of kinetic frustration [33, 34], the realization of artificial magnetic fields and topological band structures [13, 35–42] as well as the manipulation of spin-orbit coupling for cold atoms[43, 44].

For closed systems, periodically driven quantum dynamics is governed by a time-periodic Hamiltonian $H(t+T) = H(t)$ leading to an unitary time-evolution operator $U(t, 0) = \mathcal{T}e^{-\frac{i}{\hbar}\int_0^t H(\tau)d\tau}$. Because $H(t+T) = H(t)$, it is convenient to define a time-independent Hamiltonian H_F satisfying $e^{-\frac{i}{\hbar}H_F T} = U(T, 0)$. By the use of H_F , we can rewrite $U(t, 0) = U(t, 0)e^{\frac{i}{\hbar}H_F t}e^{-\frac{i}{\hbar}H_F t}$. Defining $P(t) \equiv U(t, 0)e^{\frac{i}{\hbar}H_F t}$, we can easily check that $P(t+T) = P(t)$, where $P(t)$ is an operator describing micromotion as mentioned above. H_F is the effective Floquet Hamiltonian. Having H_F , we can use its eigenstate as a basis $\{|\hbar\omega_F^{(n)}\rangle\}$. For any initial state, we can expand it with these bases as $|\Psi(0)\rangle = \sum_n c_n |\hbar\omega_F^{(n)}\rangle$, then the state at time t takes, $|\Psi(t)\rangle = \sum_n c_n e^{-i\omega_F^{(n)}t} |\Phi(t)^{(n)}\rangle$, where $|\Phi(t)^{(n)}\rangle = P(t)|\hbar\omega_F^{(n)}\rangle$ called Floquet modes. In the large

frequency limit, we can consider stroboscopic time evolution only, and the problem is then simplified to calculate H_F . For some special cases, we can get an explicit expression for H_F , but in general we can only get an approximating result for H_F [45].

In most realistic situations, a quantum system should be considered as an open quantum system coupled to an environment that induces decoherence and dissipation. Such systems are of interest for studies due to its connection to quantum computation, precision measurements and theories of quantum measurement. However, a general theory for open quantum systems similar to the Floquet theorem for closed systems remains unexplored.

In this work, we present a Floquet theorem for open systems. We find that the linear map which transforms the system from initial states to final states can be decomposed into two parts. The first part stems from the periodic time dependence of the driven system and is called micromotion, while the second contribution, which leads to deviations from the periodic evolution, originates from a time-independent Lindbladian. Using the Floquet theorem of open system, we calculated the dynamics of a dissipative two-level system with periodic Hamiltonian or Lindblad operators. We define a fidelity to quantify the deviation of the exact dynamics to that by the Floquet theorem, and find numerically the dependence of the fidelity on the driven frequency and decay rate etc..

II. FORMALISM

We start with a time dependent master equation with time-dependent Lindbladian $\mathcal{L}(t)$,

$$\partial_t \rho(t) = \mathcal{L}(t)(\rho(t)). \quad (1)$$

As $\mathcal{L}(t)$ is a linear operator, we can define a linear map $\mathcal{V}(t)$ by,

$$\partial_t \mathcal{V}(t, t_1) = \mathcal{L}(t)\mathcal{V}(t, t_1), \quad (2)$$

so the solution of the Eq.(1) is formally given by,

$$\rho(t_2) = \mathcal{V}(t_2, t_1)(\rho(t_1)), \quad (3)$$

where the propagator $\mathcal{V}(t_2, t_1)$ takes, $\mathcal{V}(t_2, t_1) = e^{\mathcal{L}(t_2)\delta t_n} \dots e^{\mathcal{L}(t_i)\delta t_i} \dots e^{\mathcal{L}(t_1)\delta t_1}$, and by the divisibility

condition, we have

$$\mathcal{V}(t_2, t_1) = \mathcal{V}(t_2, t_0)\mathcal{V}(t_0, t_1). \quad (4)$$

If $\mathcal{L}(t)$ periodically depends on time and the dynamics is Markovian, t_1 and t_2 in $\mathcal{V}(t_2, t_1)$ are not independent, i.e., the propagator depends only on $t_2 - t_1$. Noting that at each time instance, there is a infinitesimal propagator, $e^{\mathcal{L}(t_i)\delta t_i}$ and $\mathcal{L}(t) = \mathcal{L}(t + T)$, we can divide the time evolution from t_1 to t_2 into three part, i.e., starting part, middle part and ending part. In order to find the three parts, we first recall that,

$$\mathcal{V}(t_2, t_1) = \mathcal{V}(t_2 + T, t_1 + T), \quad (5)$$

then we can write the propagator in a compact form,

$$\begin{aligned} \mathcal{V}(t_2, t_1) &= \mathcal{V}(t_2, t_0 + nT)\mathcal{V}(t_0 + nT, t_0)\mathcal{V}(t_0, t_1), \\ &= \mathcal{V}(t_2, t_0 + nT)e^{n\mathcal{L}_F[t_0]T}\mathcal{V}(t_0, t_1), \\ &= \mathcal{V}(t_2, t_0 + nT)e^{-\mathcal{L}_F[t_0]\delta t_2}e^{\mathcal{L}_F[t_0](t_2-t_1)}e^{\mathcal{L}_F[t_0]\delta t_1}\mathcal{V}(t_0, t_1), \\ &= \mathcal{K}(\delta t_2)e^{\mathcal{L}_F[t_0](t_2-t_1)}\mathcal{J}(\delta t_1), \end{aligned} \quad (6)$$

where $\mathcal{K}(t) = \mathcal{V}(t_0 + t, t_0)e^{-\mathcal{L}_F[t_0]t}$, $\mathcal{J}(t) = e^{\mathcal{L}_F[t_0]t}\mathcal{V}(t_0, t_0 + t)$, $\mathcal{V}(t_0 + T, t_0) = e^{\mathcal{L}_F[t_0]T}$, $\delta t_2 = t_2 - (nT + t_0)$ and $\delta t_1 = t_1 - t_0$. $\mathcal{L}_F[t_0]$ will be referred to effective generator in later discussion. The form of $\mathcal{L}_F[t_0]$ depends on the algebraic structure of $\mathcal{L}(t)$. We use the argument t_0 to denote the dependence of \mathcal{L}_F on the starting time t_0 . Different starting time corresponds to different $\mathcal{V}(t_0 + T, t_0)$. Thus for each t_0 , we would have a set $\mathcal{L}_{F-ALL}[t_0] = \{\mathcal{L}_F[t_0]|\mathcal{V}(t_0 + T, t_0) = e^{\mathcal{L}_F[t_0]T}\}$. For different t_0 , the set is different.

In practice, to study the dynamics of an open system, we do not need to find all sets $\mathcal{L}_{F-ALL}[t_0]$: one element in $\mathcal{L}_{F-ALL}[t_0]$ is fine. For a time-dependent generator $\mathcal{L}(t)$, Magnus proposed a method to find the approximate solution for \mathcal{L}_F [12, 47], a high-frequency expansion for \mathcal{L}_F can be found using the Baker-Campbell-Hausdorff lemma[45], giving the middle part mentioned earlier.

As to the other two parts in the propagator, we can verify that $\mathcal{K}(t)$ and $\mathcal{J}(t)$ periodically depend on time, namely

$$\begin{aligned} \mathcal{K}(t + T) &= \mathcal{V}(t_0 + t + T, t_0)e^{-\mathcal{L}_F[t_0](t+T)}, \\ &= \mathcal{V}(t_0 + t + T, t_0 + T)\mathcal{V}(t_0 + T, t_0)e^{-\mathcal{L}_F[t_0](t+T)}, \\ &= \mathcal{K}(t), \end{aligned} \quad (7)$$

for $\mathcal{J}(t)$ the same proof works. By the definition of $\mathcal{K}(t)$ and $\mathcal{J}(t)$, it is easy to find that

$$\partial_t \mathcal{K}(t) = \mathcal{L}(t + t_0)\mathcal{K}(t) - \mathcal{K}(t)\mathcal{L}_F[t_0], \quad (8)$$

$$\partial_t \mathcal{J}(t) = \mathcal{L}_F[t_0]\mathcal{J}(t) - \mathcal{J}(t)\mathcal{L}(t + t_0), \quad (9)$$

Clearly the form of $\mathcal{K}(t)$ and $\mathcal{J}(t)$ depends on the choice of t_0 and $\mathcal{L}_F[t_0]$.

When we choose t_0 and $\mathcal{L}_F[t_0]$ to satisfy,

$$\mathcal{V}(t_0 + T, t_0) = e^{\mathcal{L}_F[t_0]T}, \quad (10)$$

the starting and ending parts, i.e., $\mathcal{K}(t)$ and $\mathcal{J}(t)$ can be established. By the definition of \mathcal{L}_F in Eq. (10), we find that \mathcal{L}_F itself can give a propagator for the evolution time T . In other words, given an initial state (at time t_0) of an open system, \mathcal{L}_F itself can map the initial state to the final state at time $t_0 + T$. One may wonder, how can we know the state of the system at a middle time, say $t_0 < t < t_0 + T$? Eq. (7) shows that $\mathcal{K}(t)$ and $\mathcal{J}(t)$ would help.

Now we show how to calculate \mathcal{L}_F . Without loss of generality, we set $t_1 = t_0 = 0$, so $\mathcal{J}(\delta t_1) = 1$. By the use of Magnus expansion, we can derive an expression for \mathcal{L}_F from Eq.(10). This method is available at high driving frequency, but it breaks down at low frequency. To find a \mathcal{L}_F satisfying Eq.(10), we write

$$\mathcal{V}(t) = e^{\Phi(t)}. \quad (11)$$

Obviously, $\mathcal{L}_F = \Phi(T)/T$ satisfies Eq.(10). We can obtain an expansion for $\Phi(T)$ by Magnus expansion,

$$\Phi(T) = \sum_{n=0}^{\infty} \Phi^{(n)}(T). \quad (12)$$

Similarly, $\mathcal{L}_F^{(n)} = \Phi^{(n)}(T)/T$. The first three terms are,

$$\begin{aligned} \mathcal{L}_F^{(0)} &= 1/T \int_0^T \mathcal{L}(t)dt, \\ \mathcal{L}_F^{(1)} &= 1/(2T) \int_0^T dt_1 \int_0^{t_1} dt_2 [\mathcal{L}(t_1), \mathcal{L}(t_2)], \\ \mathcal{L}_F^{(2)} &= 1/(6T) \int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \\ &\quad \{[\mathcal{L}(t_1), [\mathcal{L}(t_2), \mathcal{L}(t_3)]] + [[\mathcal{L}(t_1), \mathcal{L}(t_2)], \mathcal{L}(t_3)]\}, \end{aligned} \quad (13)$$

High-order terms can be obtained by recursion not presented here. Because $\mathcal{K}(t)$ and $\mathcal{L}(t)$ periodically depend on time, we can expand them into Fourier series,

$$\mathcal{K}(t) = \sum_{m=-\infty}^{\infty} \mathcal{K}_m e^{i\omega m t}, \quad (14)$$

$$\mathcal{L}(t) = \sum_{m=-\infty}^{\infty} \mathcal{L}_m e^{i\omega m t}. \quad (15)$$

Substituting these equations into Eq.(8), we find

$$i\omega m \mathcal{K}_m = \sum_n \mathcal{L}_n \mathcal{K}_{m-n} - \mathcal{K}_m \mathcal{L}_F. \quad (16)$$

Eq.(14) and Eq.(15) need to be truncated in order to solve Eq.(16).

Notice that \mathcal{L} would drive any initial states into a Floquet steady state defined by $\rho_F = e^{\mathcal{L}_F T}(\rho(0))$, though sometimes we can not use Magnus expansion to get \mathcal{L}_F , but we always know $\mathcal{L}_F(\rho_F) = 0$ with $t \rightarrow \infty$. So using Eq.(16), we obtain

$$i\omega m \rho_m = \sum_n \mathcal{L}_n(\rho_{m-n}), \quad (17)$$

where ρ_m is the Fourier coefficients of final state, i.e.

$$\begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & \mathcal{L}_0 + i\omega & \mathcal{L}_{-1} & \mathcal{L}_{-2} & \ddots \\ \ddots & \mathcal{L}_1 & \mathcal{L}_0 & \mathcal{L}_{-1} & \ddots \\ \ddots & \mathcal{L}_2 & \mathcal{L}_1 & \mathcal{L}_0 - i\omega & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \rho_{-1} \\ \rho_0 \\ \rho_1 \\ \vdots \end{bmatrix} = 0, \quad (18)$$

For closed systems, the time-dependent Lindbladian takes $\mathcal{L}(t)(\cdot) = -i[H(t), (\cdot)]$, Eq.(1) then returns to the quantum Liouville equation $\partial_t \rho(t) = -i[H(t), \rho(t)]$, where $H(t)$ is the system Hamiltonian. In this case, consider an infinitesimal time δt , $e^{\mathcal{L}(t)\delta t}(\cdot)$ can be written as $e^{-iH(t)\delta t}(\cdot)e^{iH(t)\delta t}$. So $\mathcal{V}(t, 0)(\cdot) = e^{-iH(t)\delta t} \dots e^{-iH(t_1)\delta t} \dots e^{-iH(0)\delta t}(\cdot)e^{iH(0)\delta t} \dots e^{iH(t_1)\delta t} \dots e^{iH(t)\delta t} = U(t, 0)(\cdot)U^\dagger(t, 0)$, where $U(t, 0)$ is an unitary time-evolution operator. By the spirit mentioned, we can choose \mathcal{L}_F in the form $-i[H_F, (\cdot)]$ to satisfy Eq.(10). If $e^{-iH_F T} = U(T, 0)$, $e^{iH_F T} = U^\dagger(T, 0)$ is naturally satisfied. Note that H_F is Hermitian[48], because U^\dagger is unitary, we have $\mathcal{K}(t)(\cdot) = P(t)(\cdot)P^\dagger(t)$. These observations together lead to $\mathcal{V}(t, 0)(\cdot) = \mathcal{K}(t)e^{\mathcal{L}_F t}(\cdot) = P(t)e^{-iH_F t}(\cdot)e^{iH_F t}P^\dagger(t)$, covering the Floquet theorem for closed quantum systems.

III. EXAMPLE

In this section we illustrate our theory with two examples. Both of them describe a two-level system subject to decoherence. In the first example, the Lindblad operator is a periodic function of time, whereas in the second the Hamiltonian is periodic in time. The master equation that describes the first example takes,

$$\partial_t \rho(t) = -i[H, \rho(t)] + \mathcal{D}(t)(\rho(t)), \quad (19)$$

in which $\mathcal{D}(t)(\rho(t)) = \gamma(2A(t)\rho(t)A^\dagger(t) - \{A^\dagger(t)A(t), \rho(t)\})$ and $H = \Omega\sigma_z$, and $A(t) = \cos(\omega t)\sigma_+ + \sin(\omega t)\sigma_-$. This type of master equation is derived in [46] that can be used to describe Cooper-pair pumping. By the use of Magnus expansion, we work out the first two leading terms of \mathcal{L}_F in large frequency limit,

$$\mathcal{L}_F^{(0)}(\cdot) = -i[H, \cdot] + \gamma(\sigma_+ \cdot \sigma_- + \sigma_- \cdot \sigma_+ - I(\cdot)), \quad (20)$$

$$\mathcal{L}_F^{(1)}(\cdot) = 2i\gamma(\Omega/\omega)(\sigma_- \cdot \sigma_- - \sigma_+ \cdot \sigma_+), \quad (21)$$

and $\mathcal{L}_F = \mathcal{L}_F^{(0)} + \mathcal{L}_F^{(1)} + \mathcal{O}(1/\omega^2)$.

Fig. 1 and Fig. 2 show the difference between the averages of σ_z given by $\mathcal{L}_F^{(0)} + \mathcal{L}_F^{(1)}$ and by the exact \mathcal{L}_F . \circ markers the periodic points of time. Here the period is chosen to be $T = 2\pi/\omega$, then $\mathcal{L}(t)$ has period $T/2$. Fig.1 shows that the amplitude of oscillation of $\langle \sigma_z \rangle$ is reduced as frequency increases. From Fig.2 we find that as γ increases, the amplitude of oscillation is enhanced, although for a time-independent open system, γ is the decay rate.

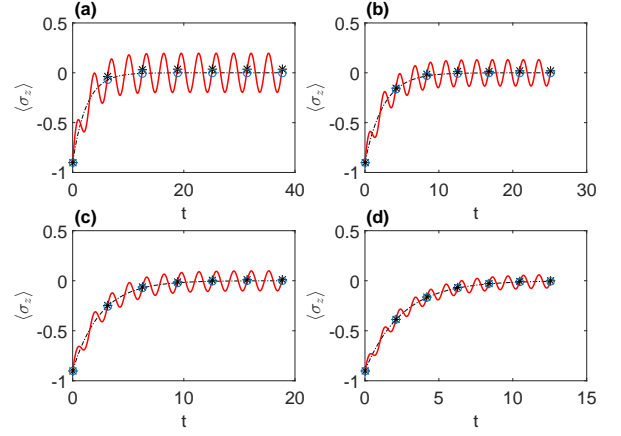


FIG. 1: (Color online) $\langle \sigma_z \rangle$ as a function of time. Parameters $\gamma = 0.2$, $\Omega = 1$ are chosen in each figure. (a)-(d) are for different driving frequency: (a) $\omega = \Omega$, (b) $\omega = 1.5\Omega$, (c) $\omega = 2\Omega$, and (d) $\omega = 3\Omega$. Red line is numerical calculation with exact $\mathcal{L}(t)$. $\langle \sigma_z \rangle$ at the period points of time are marked by \times on the line. Blue circles mark the result obtained by the approximate $\mathcal{L}_F \approx \mathcal{L}_F^{(0)} + \mathcal{L}_F^{(1)}$. The results show that as frequency increases the amplitude of oscillations decreases, and the approximate \mathcal{L}_F gets closer to the exact \mathcal{L}_F .

These results suggest that the Magnus expansions can correctly predict the asymptotical behavior of evolution, but for large γ , high-order Magnus terms should be included to get more accurate results.

The relationship between the amplitude of oscillation and the frequency might be obtained from Eq.(18). But in the high frequency limit, we can use Eq.(8) to get an well result. Indeed, for evolution time in $[0, T]$, we have the Magnus expansion $\mathcal{K}(t) = e^{\sum_{n=1}^{\infty} \Lambda^{(n)}(t)}$. Note that for every n , $\Lambda^{(n)}(t)$ preserves the periodicity. In fact, we can write $\Lambda^{(n)}(t) = \frac{1}{\omega^n} \mathcal{A}^{(n)}(t)$, where $\mathcal{A}^{(n)}(t)$ depends on the Fourier coefficient of $\mathcal{L}(t)$ [45, 47]. Then we have $\mathcal{K}(t) \approx 1 + \frac{1}{\omega} \mathcal{A}^{(1)}(t)$ and $\frac{1}{\omega} \mathcal{A}^{(1)}(t) = \frac{1}{\omega} (\int_0^t \mathcal{L}(\tau) d\omega\tau - \omega t \mathcal{L}_F^{(0)})$. When $\mathcal{L}(t) = \mathcal{F}(\omega t, \alpha, \beta, \dots)$, the amplitude of oscillation $\sim \frac{1}{\omega}$.

In the second model, we consider a spin- $\frac{1}{2}$ in a time-dependent magnetic field. The master equation is then,

$$\partial_t \rho(t) = -i[H(t), \rho(t)] + \mathcal{D}(\rho(t)), \quad (22)$$

with $H(t) = 1/2\alpha \vec{B}(t) \vec{\sigma}$, $\vec{B}(t+T) = \vec{B}(t)$ and $\mathcal{D}(\rho(t)) = \gamma(2\sigma_- \rho(t) \sigma_+ - \sigma_+ \sigma_- \rho(t) - \rho(t) \sigma_+ \sigma_-)$. The first two leading terms of \mathcal{L}_F are,

$$\mathcal{L}_F^{(0)}(\cdot) = -i[\vec{H}(t), \cdot] + \mathcal{D}(\cdot), \quad (23)$$

$$\begin{aligned} \mathcal{L}_F^{(1)}(\cdot) = & (i/2)\alpha^2 [M_x \sigma_x + M_y \sigma_y + M_z \sigma_z, \cdot] \\ & + (i/2)\alpha\gamma(N_x + iN_y)(2\sigma_- \cdot \sigma_z + \{\cdot, \sigma_-\}) \\ & - (i/2)\alpha\gamma(N_x - iN_y)(2\sigma_z \cdot \sigma_+ + \{\cdot, \sigma_+\}), \end{aligned} \quad (24)$$

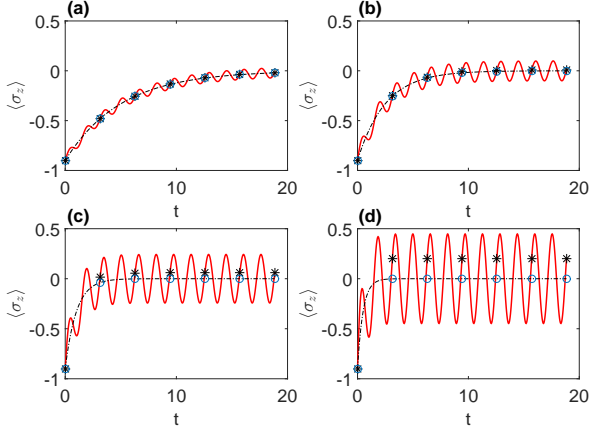


FIG. 2: (Color online) $\langle \sigma_z \rangle$ as a function of time. The lines are plotted with $\omega = 2\Omega$, $\Omega = 1$. (a)-(d) are for different γ , namely (a) $\gamma = 0.1$, (b) $\gamma = 0.2$, (c) $\gamma = 0.5$, and (d) $\gamma = 1$. Red line is the numerical result with exact $\mathcal{L}(t)$. The value at the period points in time are marked by \times . Blue circles mark the result numerically obtained by the approximate $\mathcal{L}_F \approx \mathcal{L}_F^{(0)} + \mathcal{L}_F^{(1)}$. Because terms $\mathcal{L}_F^{(n)}$ that are high order in $\frac{1}{\omega}$ are not zero and depend on γ . So for larger γ , $\mathcal{L}_F - (\mathcal{L}_F^{(0)} + \mathcal{L}_F^{(1)})$ no more approaches zero, as shown by the deviation of blue circle from the \times .

where

$$\begin{aligned} M_x &= \overline{\overline{B_y(t_2)B_z(t_1) - B_y(t_1)B_z(t_2)}} \\ M_y &= \overline{\overline{B_z(t_2)B_x(t_1) - B_z(t_1)B_x(t_2)}} \\ M_z &= \overline{\overline{B_x(t_2)B_y(t_1) - B_x(t_1)B_y(t_2)}}, \end{aligned}$$

and

$$\begin{aligned} N_x &= \overline{\overline{B_x(t_1) - B_x(t_2)}} \\ N_y &= \overline{\overline{B_y(t_1) - B_y(t_2)}}, \end{aligned}$$

here $\overline{\overline{f(t_1, t_2)}} = 1/(2T) \int_0^T dt_1 \int_0^{t_1} dt_2 f(t_1, t_2)$.

If we consider \vec{B} rotates around an axis with spherical coordinate (θ, φ) , and the angle of deviation from the z-axis is β , i.e. $\vec{B}(t) = \vec{B}_p + \vec{B}_v(t)$, $\vec{B}_p = \cos(\beta)(\cos(\theta)\sin(\varphi), \sin(\theta)\sin(\varphi), \cos(\varphi))$ and $\vec{B}_v(t) = \sin(\beta)(\sin(\omega t)(\cos(\theta)\cos(\varphi), \sin(\theta)\cos(\varphi), -\sin(\varphi)) + \cos(\omega t)(\sin(\theta), -\cos(\theta), 0))$, we have,

$$\begin{aligned} M_x &= 1/2\omega \sin\beta(\sin\beta \cos\theta \sin\varphi + 2\cos\beta \sin\theta) \\ M_y &= 1/2\omega \sin\beta(\sin\beta \sin\theta \sin\varphi - 2\cos\beta \cos\theta) \\ M_z &= 1/2\omega \cos\varphi \sin^2\beta \\ N_x &= -1/\omega \cos\theta \cos\varphi \sin\beta \\ N_y &= -1/\omega \sin\theta \cos\varphi \sin\beta \\ \overline{H(t)} &= (1/2)\alpha \cos\beta[\cos\varphi \sigma_z + \sin\varphi(\cos\theta \sigma_x + \sin\theta \sigma_y)]. \end{aligned}$$

As Eq. (25) shows, the first order of the effective generator includes three parts. The first part behaves like an effective Hamilton $\frac{1}{2}\alpha^2 \mathbf{M}\sigma$. And \mathbf{M} is a function of $\theta, \varphi, \beta, \omega$,

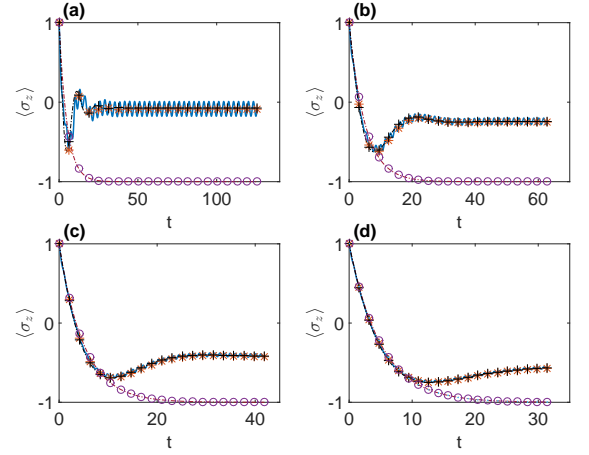


FIG. 3: (Color online) The average of σ_z versus time. Parameter $\theta = \frac{\pi}{4}$, $\varphi = \frac{\pi}{2}$, $\beta = \frac{\pi}{2}$, $\alpha = 1$, and $\gamma = 0.1$ are chosen for these lines. $\omega = 1, 2, 3, 4$ are for (a), (b), (c), and (d), respectively. Blue line is the numerical result by exact $\mathcal{L}(t)$. The values at the period points in time are marked by \times . Purple circles mark the result gotten by the approximate \mathcal{L}_F up to zeroth order. + marks the result obtained by the approximate \mathcal{L}_F up to the order in the expansion of $\mathcal{L}(t)$.

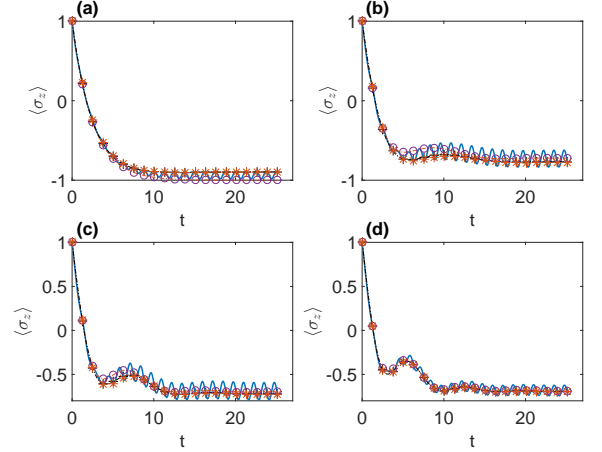


FIG. 4: (Color online) The average of σ_z versus time. Parameters chosen are $\omega = 5$, $\theta = \frac{\pi}{4}$, $\varphi = \frac{\pi}{4}$, $\alpha = 1$, and $\gamma = 0.2$. $\beta = \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}$, and $\frac{\pi}{8}$ for (a), (b), (c), and (d), respectively. Blue line is the numerical calculation with exact $\mathcal{L}(t)$. The period points in time are marked by \times . Purple circles mark the result by the approximate \mathcal{L}_F up to the zeroth order (first term only) in the expansion. + marks the result by the approximate \mathcal{L}_F up to the first order (first term and second term) in the expansion.

$\mathbf{M} = \frac{1}{\omega} \mathbf{m}(\theta, \varphi, \beta)$. The second and third terms are inversely proportional to $\frac{1}{\omega}$. For a special case of $\theta = \frac{\pi}{4}, \varphi = \frac{\pi}{2}$ and $\beta = \frac{\pi}{2}$, terms with any one of $\overline{H(t)}, N_x, N_y$ or M_z vanish. In this case if γ is smaller than α^2 , $\mathcal{L}_F^{(1)}(\cdot)$ would dominate the dynamics. To this extend, we can say the system is sensitive to the frequency of the driving B , because $\mathcal{L}_F^{(1)}(\cdot) \sim 1/\omega$. See Fig.3 and Fig.4.

IV. CONCLUSION AND DISCUSSIONS

We have derived a general approach to solve periodically driven systems subject to decoherence, which we call the Floquet theorem of open system. The theorem allows to obtain an effective time-independent Lindbladian for different driving regimes. We show that in the high-frequency limit, the leading-order of the Mangus expansion agrees well with the exact dynamics. When we generalize the results to an open system with two periods, say period T_1 and T_2 , higher order expansion in $\frac{1}{\omega}$ than a system with only T_1 or T_2 periodicity is necessary. The reason is as follows. Consider $T_1/T_2 \approx p/q$,

p, q are prime number, the overall period of the system becomes $T_2 \text{lcm}(p, q)/q$ (with lcm denoting the lowest common multiple), which usually is much bigger than T_1 and T_2 . So, higher order Mangus expansion has to be taken into account.

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